TABLE OF LAPLACE TRANSFORMS (*continued*)

	$\bar{f}(s)$	$f(t)$
28	$\dfrac{3a^2}{s^3+a^3}$	$e^{-at} - e^{at/2} \times \left(\cos\dfrac{at\sqrt{3}}{2} - \sqrt{3}\sin\dfrac{at\sqrt{3}}{2}\right)$
29	$\dfrac{4a^3}{s^4+4a^4}$	$\sin at \cosh at - \cos at \sinh at$
30	$\dfrac{s}{s^4+4a^4}$	$\dfrac{1}{2a^2}\sin at \sinh at$
31	$\dfrac{1}{s^4-a^4}$	$\dfrac{1}{2a^3}(\sinh at - \sin at)$
32	$\dfrac{s}{s^4-a^4}$	$\dfrac{1}{2a^2}(\cosh at - \cos at)$
33	$\dfrac{8a^3s^2}{(s^2+a^2)^3}$	$(1+a^2t^2)\sin at - at\cos a$
34*	$\dfrac{1}{s}\left(\dfrac{s-1}{s}\right)^n$	$L_n(t) = \dfrac{e^t}{n!}\dfrac{d^n}{dt^n}(t^n e^{-t})$
35	$\dfrac{s}{(s-a)^{3/2}}$	$\dfrac{1}{\sqrt{\pi t}}e^{at}(1+2at)$
36	$\sqrt{s-a} - \sqrt{s-b}$	$\dfrac{1}{2\sqrt{\pi t^3}}(e^{bt}-e^{at})$
37	$\dfrac{1}{\sqrt{s}+a}$	$\dfrac{1}{\sqrt{\pi t}} - a e^{a^2t}\operatorname{erfc}(a\sqrt{t})$
38	$\dfrac{\sqrt{s}}{s-a^2}$	$\dfrac{1}{\sqrt{\pi t}} + a e^{a^2t}\operatorname{erf}(a\sqrt{t})$
39	$\dfrac{\sqrt{s}}{s+a^2}$	$\dfrac{1}{\sqrt{\pi t}} - \dfrac{2a}{\sqrt{\pi}}e^{-a^2t}\displaystyle\int_0^{a\sqrt{t}} e^{\lambda^2}\,d\lambda$
40	$\dfrac{1}{\sqrt{s}(s-a^2)}$	$\dfrac{1}{a}e^{a^2t}\operatorname{erf}(a\sqrt{t})$
41	$\dfrac{1}{\sqrt{s}(s+a^2)}$	$\dfrac{2}{a\sqrt{\pi}}e^{-a^2t}\displaystyle\int_0^{a\sqrt{t}} e^{\lambda^2}\,d\lambda$

* $L_n(t)$ is the Laguerre polynomial of degree n.

TABLE OF LAPLACE TRANSFORMS (*continued*)

	$\bar{f}(s)$	$f(t)$
42	$\dfrac{b^2-a^2}{(s-a^2)(b+\sqrt{s})}$	$e^{a^2t}[b - a\operatorname{erf}(a\sqrt{t})] - b e^{b^2t}\operatorname{erfc}(b\sqrt{t})$
43	$\dfrac{1}{\sqrt{s}(\sqrt{s}+a)}$	$e^{a^2t}\operatorname{erfc}(a\sqrt{t})$
44	$\dfrac{1}{(s+a)\sqrt{s+b}}$	$\dfrac{1}{\sqrt{b-a}}e^{-at}\operatorname{erf}(\sqrt{b-a}\sqrt{t})$
45	$\dfrac{b^2-a^2}{\sqrt{s}(s-a^2)(\sqrt{s}+b)}$	$e^{a^2t}\left[\dfrac{b}{a}\operatorname{erf}(a\sqrt{t})-1\right] + e^{b^2t}\operatorname{erfc}(b\sqrt{t})$
46*	$\dfrac{(1-s)^n}{s^{n+\frac{1}{2}}}$	$\dfrac{n!}{(2n)!\sqrt{\pi t}}H_{2n}(\sqrt{t})$
47	$\dfrac{(1-s)^n}{s^{n+\frac{3}{2}}}$	$-\dfrac{n!}{\sqrt{\pi}(2n+1)!}H_{2n+1}(\sqrt{t})$
48†	$\dfrac{\sqrt{s+2a}}{\sqrt{s}} - 1$	$a e^{-at}[I_1(at)+I_0(at)]$
49	$\dfrac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{-\frac{1}{2}(a+b)t}I_0\left(\dfrac{a-b}{2}t\right)$
50	$\dfrac{\Gamma(k)}{(s+a)^k(s+b)^k}\ (k>0)$	$\sqrt{\pi}\left(\dfrac{t}{a-b}\right)^{k-\frac{1}{2}} e^{-\frac{1}{2}(a+b)t} \times I_{k-\frac{1}{2}}\left(\dfrac{a-b}{2}t\right)$
51	$\dfrac{1}{(s+a)^{\frac{1}{2}}(s+b)^{\frac{3}{2}}}$	$t e^{-\frac{1}{2}(a+b)t}\left[I_0\left(\dfrac{a-b}{2}t\right) + I_1\left(\dfrac{a-b}{2}t\right)\right]$
52	$\dfrac{\sqrt{s+2a}-\sqrt{s}}{\sqrt{s+2a}+\sqrt{s}}$	$\dfrac{1}{t}e^{-at}I_1(at)$
53	$\dfrac{(a-b)^k}{(\sqrt{s+a}+\sqrt{s+b})^{2k}}\ (k>0)$	$\dfrac{k}{t}e^{-\frac{1}{2}(a+b)t}I_k\left(\dfrac{a-b}{2}t\right)$

* $H_n(x)$ is the Hermite polynomial, $H_n(x) = e^{x^2}\dfrac{d^n}{dx^n}(e^{-x^2})$.

† $I_n(x) = i^{-n}J_n(ix)$, where J_n is Bessel's function of the first kind.

TABLE OF LAPLACE TRANSFORMS (*continued*)

	$\bar{f}(s)$	$f(t)$
54	$\dfrac{(\sqrt{s+a}+\sqrt{s})^{-2\nu}}{\sqrt{s}\sqrt{s+a}}\ (\nu > -1)$	$\dfrac{1}{a^{\nu}} e^{-\frac{1}{2}at} I_{\nu}(\tfrac{1}{2}at)$
55	$\dfrac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$.
56	$\dfrac{(\sqrt{s^2+a^2}-s)^{\nu}}{\sqrt{s^2+a^2}}\ (\nu > -1)$	$a^{\nu} J_{\nu}(at)$
57	$\dfrac{1}{(s^2+a^2)^k}\ (k > 0)$	$\dfrac{\sqrt{\pi}}{\Gamma(k)}\left(\dfrac{t}{2a}\right)^{k-\frac{1}{2}} J_{k-\frac{1}{2}}(at)$
58	$(\sqrt{s^2+a^2}-s)^k\ (k > 0)$	$\dfrac{ka^k}{t} J_k(at)$
59	$\dfrac{(s-\sqrt{s^2-a^2})^{\nu}}{\sqrt{s^2-a^2}}\ (\nu > -1)$	$a^{\nu} I_{\nu}(at)$
60	$\dfrac{1}{(s^2-a^2)^k}\ (k > 0)$	$\dfrac{\sqrt{\pi}}{\Gamma(k)}\left(\dfrac{t}{2a}\right)^{k-\frac{1}{2}} I_{k-\frac{1}{2}}(at)$
61	$\dfrac{e^{-ks}}{s}$	$S_k(t) = \begin{cases} 0 & \text{when } 0 < t < k \\ 1 & \text{when } t > k \end{cases}$
62	$\dfrac{e^{-ks}}{s^2}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ t-k & \text{when } t > k \end{cases}$
63	$\dfrac{e^{-ks}}{s^{\mu}}\ (\mu > 0)$	$\begin{cases} 0 & \text{when } 0 < t < k \\ \dfrac{(t-k)^{\mu-1}}{\Gamma(\mu)} & \text{when } t > k \end{cases}$
64	$\dfrac{1-e^{-ks}}{s}$	$\begin{cases} 1 & \text{when } 0 < t < k \\ 0 & \text{when } t > k \end{cases}$
65	$\dfrac{1}{s(1-e^{-ks})} = \dfrac{1+\coth\frac{1}{2}ks}{2s}$	$S(k, t) = n$ when $(n-1)k < t < nk$ (Fig. 6.4)
66	$\dfrac{1}{s(e^{ks}-a)}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ 1+a+a^2+\ldots+a^{n-1} & \text{when } nk < t < (n+1)k \\ & (n = 1, 2, \ldots) \end{cases}$
67	$\dfrac{1}{s}\tanh ks$	$M(2k, t) = (-1)^{n-1}$ when $2k(n-1) < t < 2kn$ $(n = 1, 2, \ldots)$
68	$\dfrac{1}{s(1+e^{-ks})}$	$\frac{1}{2}M(k, t)+\frac{1}{2} = \dfrac{1-(-1)^n}{2}$ when $(n-1)k < t < nk$
69	$\dfrac{1}{s^2}\tanh\frac{1}{2}ks$	$H(k, t) = \begin{cases} t & \text{when } 2nk < t < (2n+1)k \\ 2k-t & \text{when } (2n+1)k < t < (2n+2)k \end{cases}$
70	$\dfrac{1}{s\sinh ks}$	$F(t) = 2(n-1)$ when $(2n-3)k < t < (2n-1)k$ $(t > 0)$
71	$\dfrac{1}{s\cosh ks}$	$M(2k, t+3k)+1 = 1+(-1)^n$ when $(2n-3)k < t < (2n-1)k$ $(t > 0)$
72	$\dfrac{1}{s}\coth ks$	$F(t) = 2n-1$ when $2k(n-1) < t < 2kn$
73	$\dfrac{k}{s^2+k^2}\coth\dfrac{\pi s}{2k}$	$\lvert\sin kt\rvert$
74	$\dfrac{1}{(s^2+1)(1-e^{-\pi s})}$	$\begin{cases} \sin t & \text{when } (2n-2)\pi < t < (2n-1)\pi \\ 0 & \text{when } (2n-1)\pi < t < 2n\pi \end{cases}$
75	$\dfrac{1}{s}e^{-(k/s)}$	$J_0(2\sqrt{kt})$
76	$\dfrac{1}{\sqrt{s}}e^{-(k/s)}$	$\dfrac{1}{\sqrt{\pi t}}\cos 2\sqrt{kt}$
77	$\dfrac{1}{\sqrt{s}}e^{k/s}$	$\dfrac{1}{\sqrt{\pi t}}\cosh 2\sqrt{kt}$
78	$\dfrac{1}{s^{\frac{3}{2}}}e^{-(k/s)}$	$\dfrac{1}{\sqrt{\pi k}}\sin 2\sqrt{kt}$
79	$\dfrac{1}{s^{\frac{3}{2}}}e^{k/s}$	$\dfrac{1}{\sqrt{\pi k}}\sinh 2\sqrt{kt}$
80	$\dfrac{1}{s^{\mu}}e^{-(k/s)}\ (\mu > 0)$	$\left(\dfrac{t}{k}\right)^{(\mu-1)/2} J_{\mu-1}(2\sqrt{kt})$
81	$\dfrac{1}{s^{\mu}}e^{k/s}\ (\mu > 0)$	$\left(\dfrac{t}{k}\right)^{(\mu-1)/2} I_{\mu-1}(2\sqrt{kt})$

TABLE OF LAPLACE TRANSFORMS (*continued*)

	$\bar{f}(s)$	$f(t)$
82	$e^{-k\sqrt{s}}(k>0)$	$\dfrac{k}{2\sqrt{\pi t^3}}\exp\left(-\dfrac{k^2}{4t}\right)$
83	$\dfrac{1}{s}e^{-k\sqrt{s}}(k\geq 0)$	$\operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)$
84	$\dfrac{1}{\sqrt{s}}e^{-k\sqrt{s}}(k\geq 0)$	$\dfrac{1}{\sqrt{\pi t}}\exp\left(-\dfrac{k^2}{4t}\right)$
85	$s^{-\frac{3}{2}}e^{-k\sqrt{s}}(k\geq 0)$	$2\sqrt{\dfrac{t}{\pi}}\exp\left(-\dfrac{k^2}{4t}\right)-k\operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)$
86	$\dfrac{a\,e^{-k\sqrt{s}}}{s(a+\sqrt{s})}(k\geq 0)$	$-e^{ak}e^{a^2t}\operatorname{erfc}\left(a\sqrt{t}+\dfrac{k}{2\sqrt{t}}\right)+\operatorname{erfc}\left(\dfrac{k}{2\sqrt{t}}\right)$
87	$\dfrac{e^{-k\sqrt{s}}}{\sqrt{s}(a+\sqrt{s})}(k\geq 0)$	$e^{ak}e^{a^2t}\operatorname{erfc}\left(a\sqrt{t}+\dfrac{k}{2\sqrt{t}}\right)$
88	$\dfrac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s(s+a)}}$	$\begin{cases}0 & \text{when } 0<t<k\\ e^{-\frac{1}{2}at}I_0(\frac{1}{2}a\sqrt{t^2-k^2}) & \text{when } t>k\end{cases}$
89	$\dfrac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$	$\begin{cases}0 & \text{when } 0<t<k\\ J_0(a\sqrt{t^2-k^2}) & \text{when } t>k\end{cases}$
90	$\dfrac{e^{-k\sqrt{s^2-a^2}}}{\sqrt{s^2-a^2}}$	$\begin{cases}0 & \text{when } 0<t<k\\ I_0(a\sqrt{t^2-k^2}) & \text{when } t>k\end{cases}$
91	$\dfrac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}}(k\geq 0)$	$J_0(a\sqrt{t^2+2kt})$
92	$e^{-ks}-e^{-k\sqrt{s^2+a^2}}$	$\begin{cases}0 \quad \text{when } 0<t<k\\ \dfrac{ak}{\sqrt{t^2-k^2}}J_1(a\sqrt{t^2-k^2}) \\ \qquad\qquad \text{when } t>k\end{cases}$
93	$e^{-k\sqrt{s^2-a^2}}-e^{-ks}$	$\begin{cases}0 \quad \text{when } 0<t<k\\ \dfrac{ak}{\sqrt{t^2-k^2}}I_1(a\sqrt{t^2-k^2}) \\ \qquad\qquad \text{when } t>k\end{cases}$

TABLE OF LAPLACE TRANSFORMS (*continued*)

	$\bar{f}(s)$	$f(t)$
94	$\dfrac{a^\nu e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}(\sqrt{s^2+a^2}+s)^\nu}(\nu>-1)$	$\begin{cases}0 \quad \text{when } 0<t<k\\ \left(\dfrac{t-k}{t+k}\right)^{\frac{1}{2}\nu}J_\nu(a\sqrt{t^2-k^2}) \\ \qquad\qquad \text{when } t>k\end{cases}$
95	$\dfrac{1}{s}\log s$	$\Gamma'(1)-\log t \quad [\Gamma'(1)=-0{\cdot}5772]$
96	$\dfrac{1}{s^k}\log s(k>0)$	$t^{k-1}\left[\dfrac{\Gamma'(k)}{[\Gamma(k)]^2}-\dfrac{\log t}{\Gamma(k)}\right]$
97*	$\dfrac{\log s}{s-a}(a>0)$	$e^{at}[\log a-\operatorname{Ei}(-at)]$
98*	$\dfrac{\log s}{s^2+1}$	$\cos t\operatorname{Si}(t)-\sin t\operatorname{Ci}(t)$
99*	$\dfrac{s\log s}{s^2+1}$	$-\sin t\operatorname{Si}(t)-\cos t\operatorname{Ci}(t)$
100*	$\dfrac{1}{s}\log(1+ks)(k>0)$	$-\operatorname{Ei}\left(-\dfrac{t}{k}\right)$
101	$\log\dfrac{s-a}{s-b}$	$\dfrac{1}{t}(e^{bt}-e^{at})$
102*	$\dfrac{1}{s}\log(1+k^2s^2)$	$-2\operatorname{Ci}\left(\dfrac{t}{k}\right)$
103*	$\dfrac{1}{s}\log(s^2+a^2)(a>0)$	$2\log a-2\operatorname{Ci}(at)$
104*	$\dfrac{1}{s^2}\log(s^2+a^2)(a>0)$	$\dfrac{2}{a}[at\log a+\sin at-at\operatorname{Ci}(at)]$
105	$\log\dfrac{s^2+a^2}{s^2}$	$\dfrac{2}{t}(1-\cos at)$
106	$\log\dfrac{s^2-a^2}{s^2}$	$\dfrac{2}{t}(1-\cosh at)$
107	$\arctan\dfrac{k}{s}$	$\dfrac{1}{t}\sin kt$

* The exponential-integral, cosine-integral, and sine-integral functions $\operatorname{Ei}(-t)$, $\operatorname{Ci}(t)$, and $\operatorname{Si}(t)$ are defined in Section 5.5.3.

TABLE OF LAPLACE TRANSFORMS (*continued*)

	$\bar{f}(s)$	$f(t)$
108	$\frac{1}{s}\arctan\frac{k}{s}$	$\mathrm{Si}\,(kt)$
109	$e^{k^2s^2}\,\mathrm{erfc}\,(ks)\,(k>0)$	$\frac{1}{k\sqrt{\pi}}\exp\left(-\frac{t^2}{4k^2}\right)$
110	$\frac{1}{s}e^{k^2s^2}\,\mathrm{erfc}\,(ks)\,(k>0)$	$\mathrm{erf}\left(\frac{t}{2k}\right)$
111	$e^{ks}\,\mathrm{erfc}\,\sqrt{ks}\,(k>0)$	$\frac{\sqrt{k}}{\pi\sqrt{t}(t+k)}$
112	$\frac{1}{\sqrt{s}}\mathrm{erfc}\,(\sqrt{ks})$	$\begin{cases}0 & \text{when } 0<t<k\\ (\pi t)^{-\frac{1}{2}} & \text{when } t>k\end{cases}$
113	$\frac{1}{\sqrt{s}}e^{ks}\,\mathrm{erfc}\,(\sqrt{ks})\,(k>0)$	$\frac{1}{\sqrt{\pi(t+k)}}$
114	$\mathrm{erf}\left(\frac{k}{\sqrt{s}}\right)$	$\frac{1}{\pi t}\sin\,(2k\sqrt{t})$
115	$\frac{1}{\sqrt{s}}e^{k^2/s}\,\mathrm{erfc}\left(\frac{k}{\sqrt{s}}\right)$	$\frac{1}{\sqrt{\pi t}}e^{-2k\sqrt{t}}$
116	$K_0(ks)$	$\begin{cases}0 & \text{when } 0<t<k\\ (t^2-k^2)^{-\frac{1}{2}} & \text{when } t>k\end{cases}$
117	$K_0(k\sqrt{s})$	$\frac{1}{2t}\exp\left(-\frac{k^2}{4t}\right)$
118	$\frac{1}{s}e^{ks}K_1(ks)$	$\frac{1}{k}\sqrt{t(t+2k)}$
119	$\frac{1}{\sqrt{s}}K_1(k\sqrt{s})$	$\frac{1}{k}\exp\left(-\frac{k^2}{4t}\right)$
120	$\frac{1}{\sqrt{s}}e^{k/s}K_0\left(\frac{k}{s}\right)$	$\frac{2}{\sqrt{\pi t}}K_0(2\sqrt{2kt})$
121	$\pi e^{-ks}I_0(ks)$	$\begin{cases}[t(2k-t)]^{-\frac{1}{2}} & \text{when } 0<t<2k\\ 0 & \text{when } t>2k\end{cases}$
122	$e^{-ks}I_1(ks)$	$\begin{cases}\frac{k-t}{\pi k\sqrt{t(2k-t)}} & \text{when } 0<t<2k\\ 0 & \text{when } t>2k\end{cases}$
123	$-e^{as}\mathrm{Ei}(-as)$	$\frac{1}{t+a}\,(a>0)$
124	$\frac{1}{a}+se^{as}\mathrm{Ei}(-as)$	$\frac{1}{(t+a)^2}\,(a>0)$
125	$\left(\frac{\pi}{2}-\mathrm{Si}\,s\right)\cos s+\mathrm{Ci}\,s\sin s$	$\frac{1}{t^2+1}$